

CCA Security

CS/ECE 407

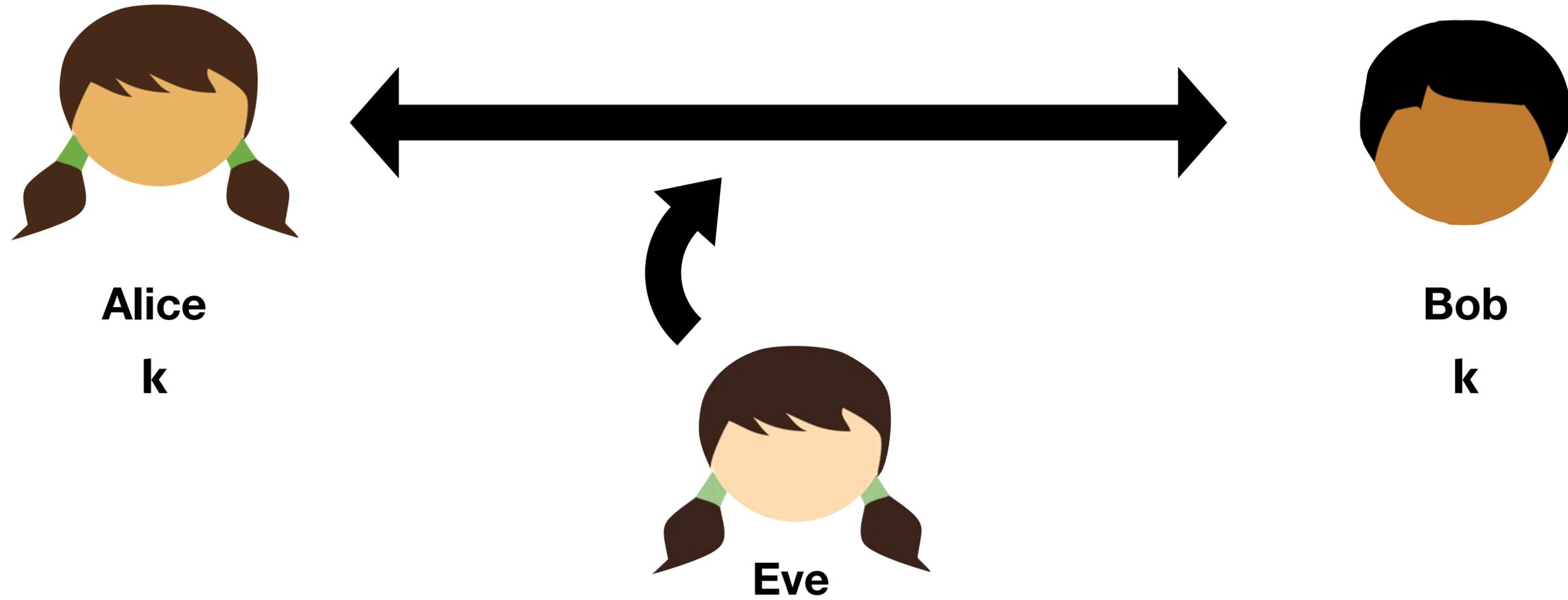
Today's objectives

See intricate attacks by an active adversary

Understand the definition of **Security against Chosen Ciphertext Attacks (CCA)**

Explain why CTR/CBC mode block cipher is not CCA secure

Connect CCA security with **malleability**



Confidentiality

Can Alice and Bob prevent Eve from listening?

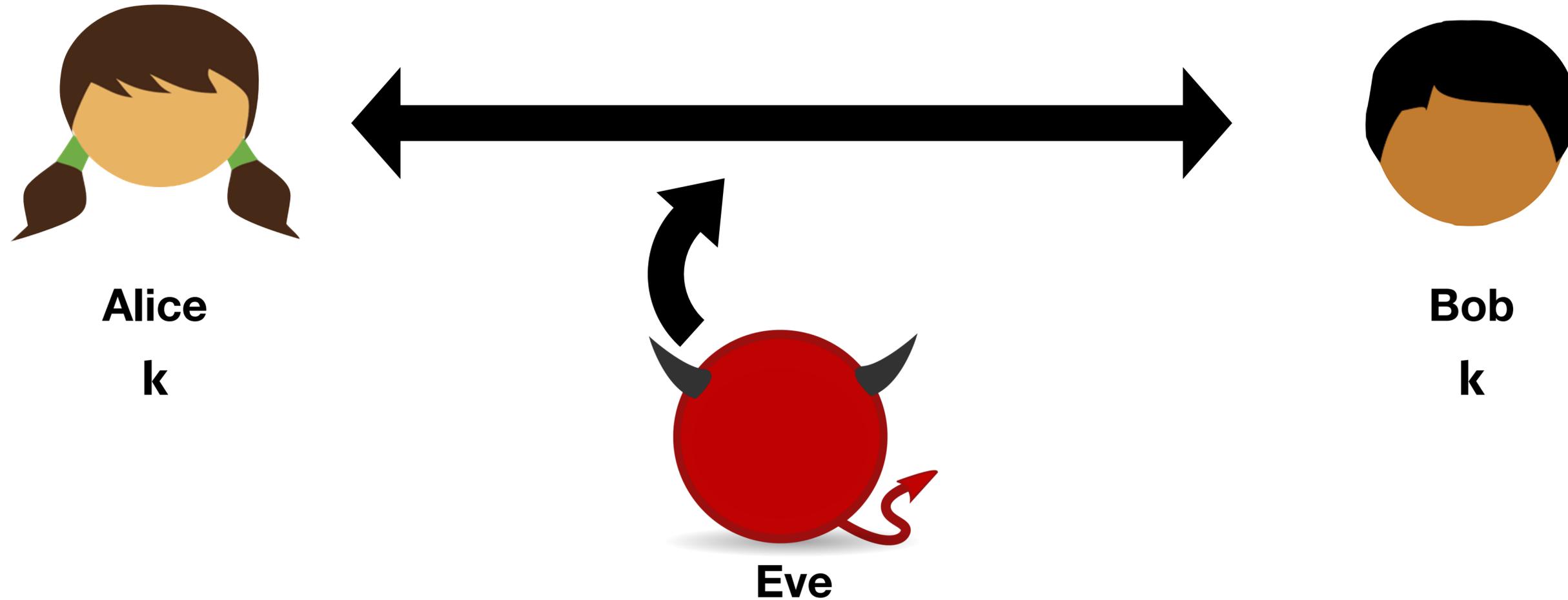
A cipher (Enc, Dec) has **ciphertext indistinguishability against a chosen plaintext attack (CPA)** if:

Let $Enc_L(k, m_0, m_1) = Enc(k, m_0)$

Let $Enc_R(k, m_0, m_1) = Enc(k, m_1)$

Where m_0, m_1 are of the same length

$$\left\{ Enc_L(k, \cdot, \cdot) \mid k \leftarrow K \right\} \approx \left\{ Enc_R(k, \cdot, \cdot) \mid k \leftarrow K \right\}$$



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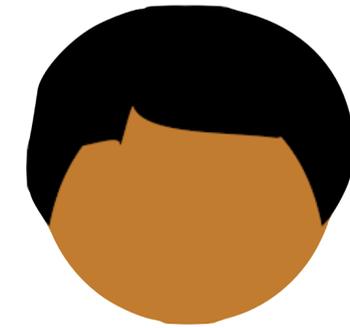
Authenticity

Can Bob be sure Eve did not send the message?

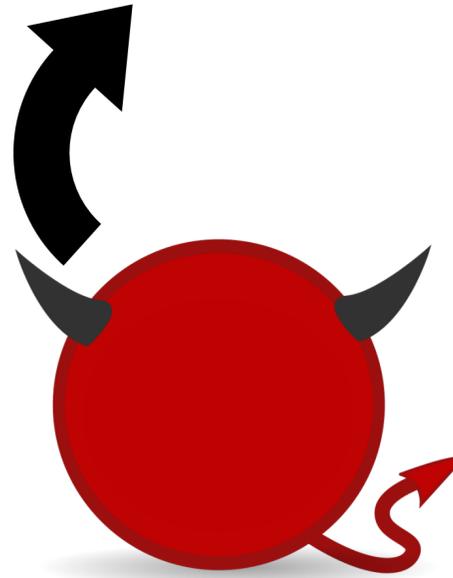
Can Bob be sure Eve did not alter a message from Alice?



Alice
k



Bob
k



Eve

Attacks by the adversary can be subtle.

Let's see how adversary can send messages to break a CPA-secure scheme

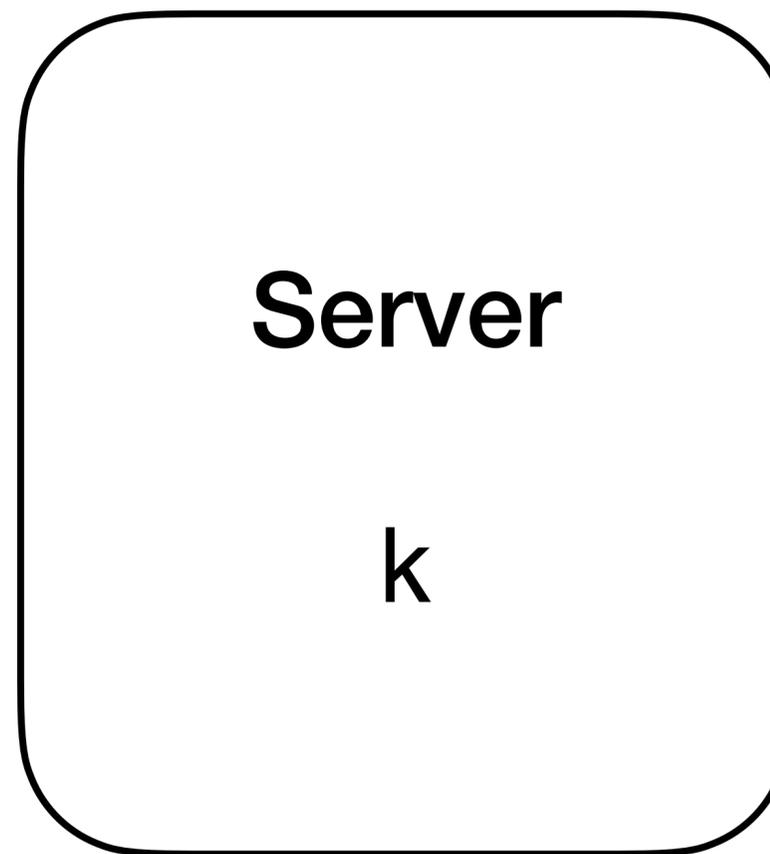
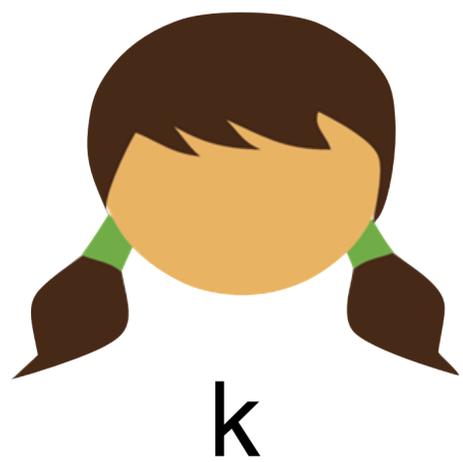
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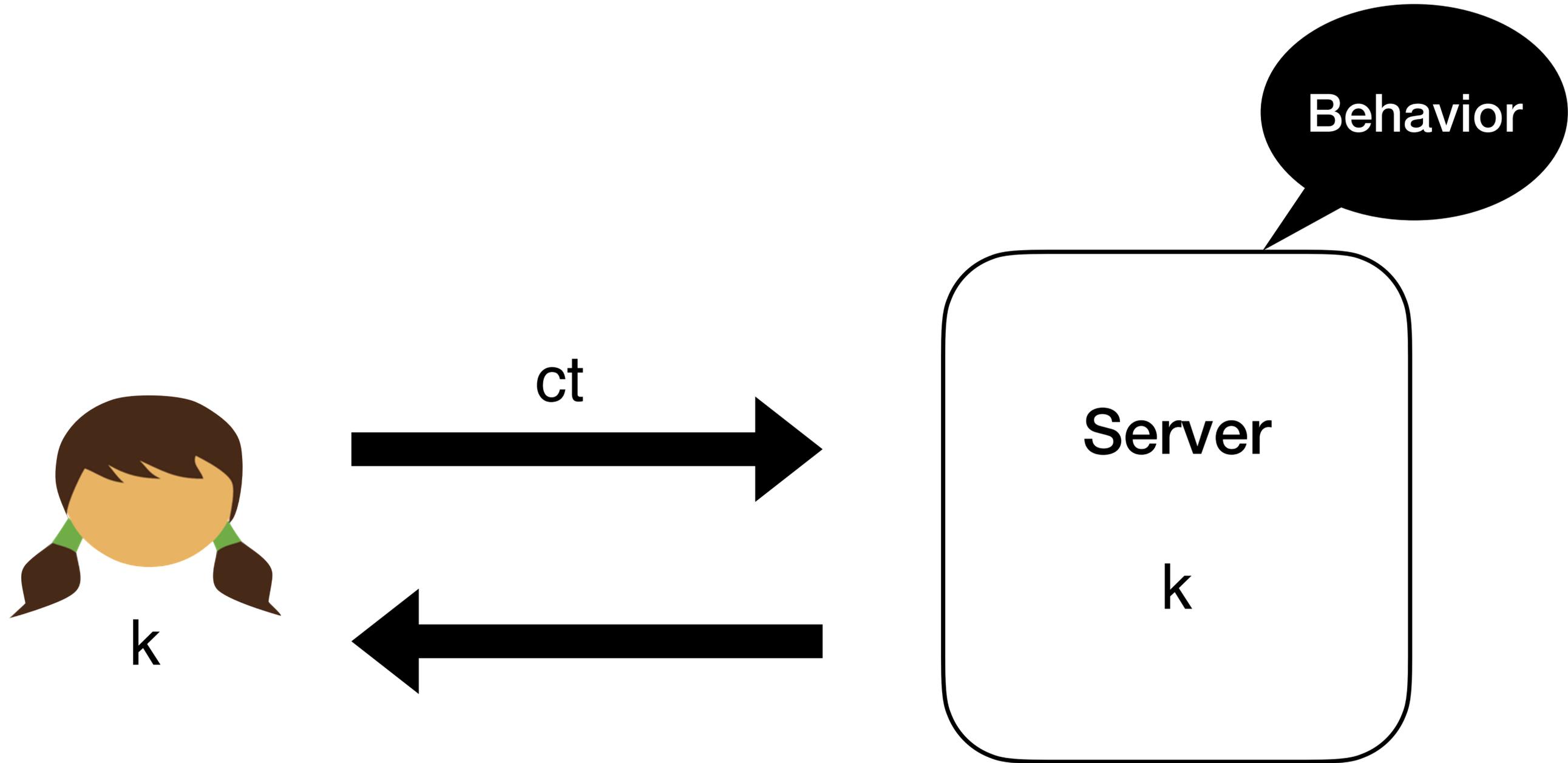
Can Alice and Bob prevent Eve from listening?

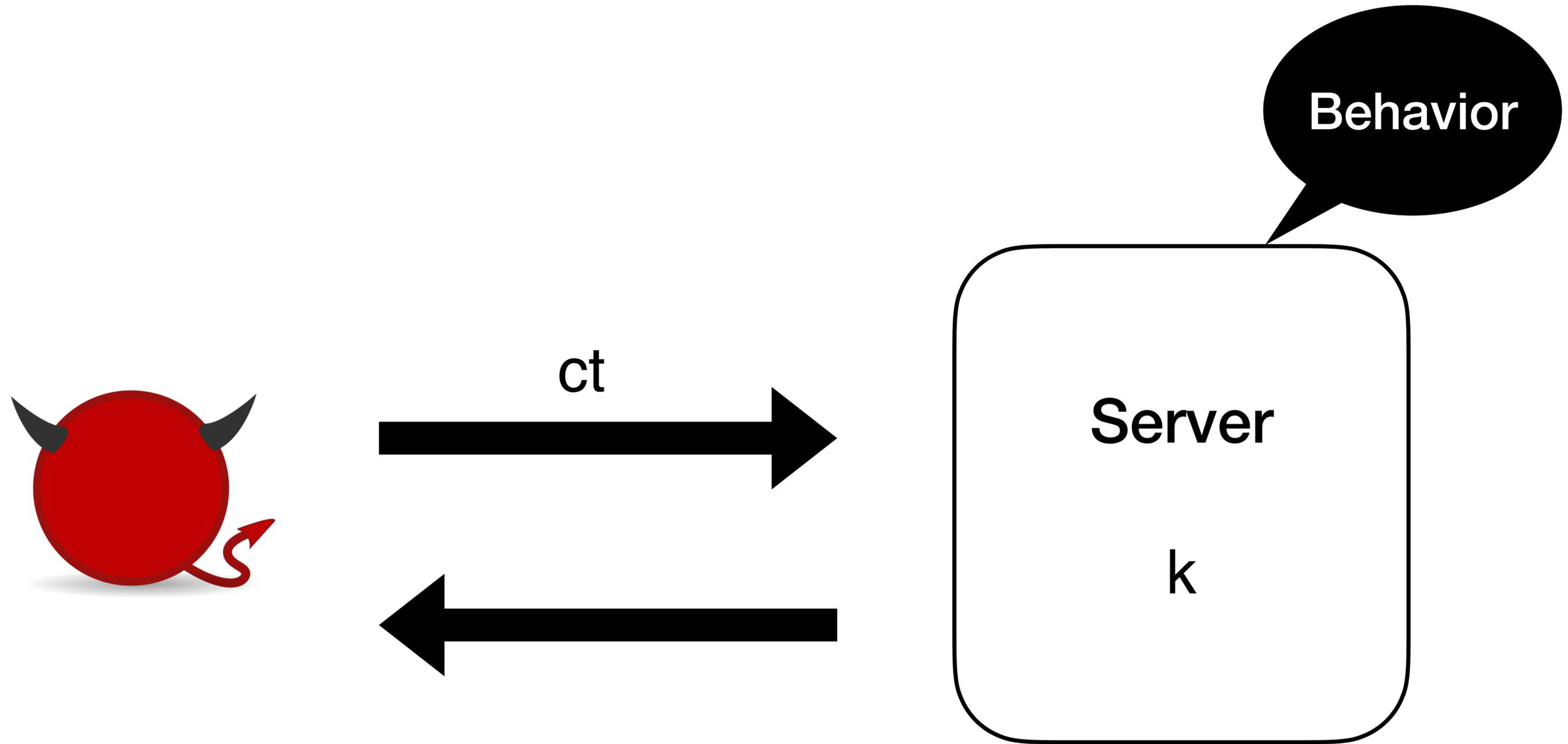
Authenticity

Can Bob be sure Eve did not send the message?

Can Bob be sure Eve did not alter a message from Alice?





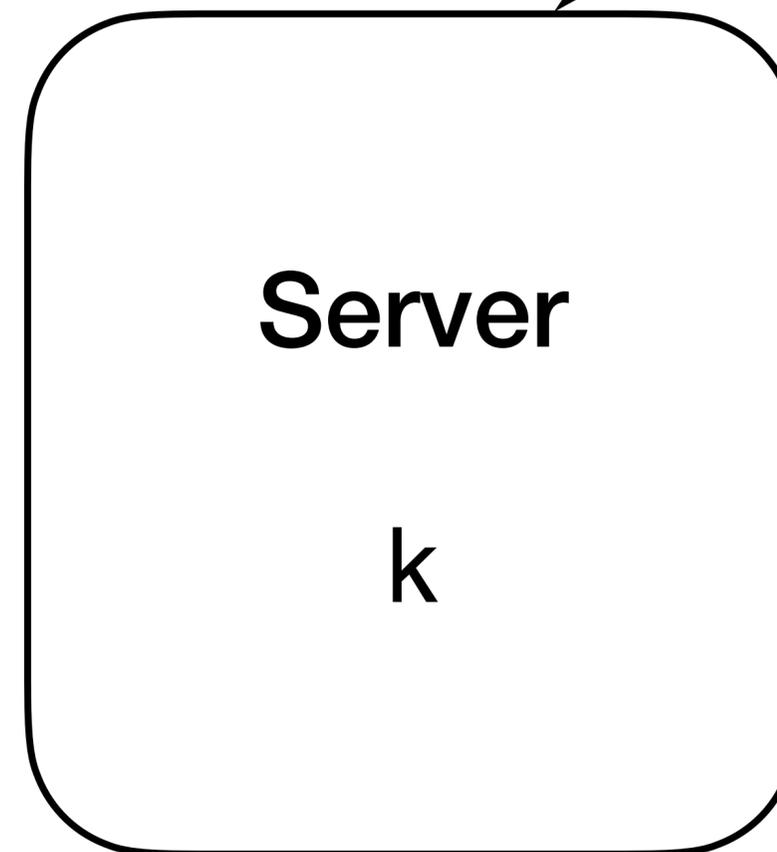
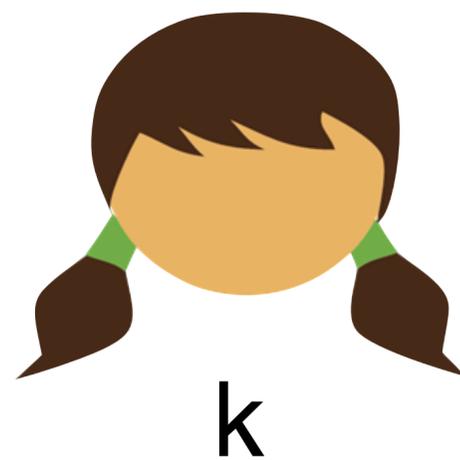


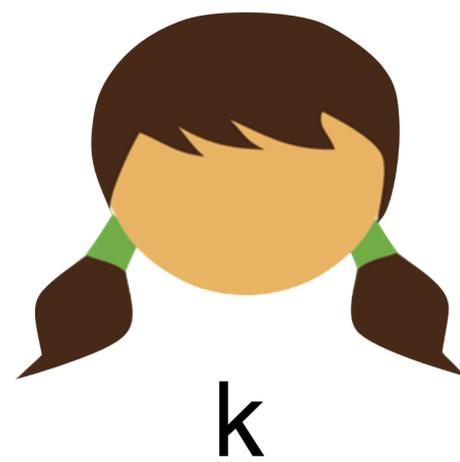
**Adversary can try to send ciphertexts,
and see what happens**

Warm-up: **Null Oracle Attack**

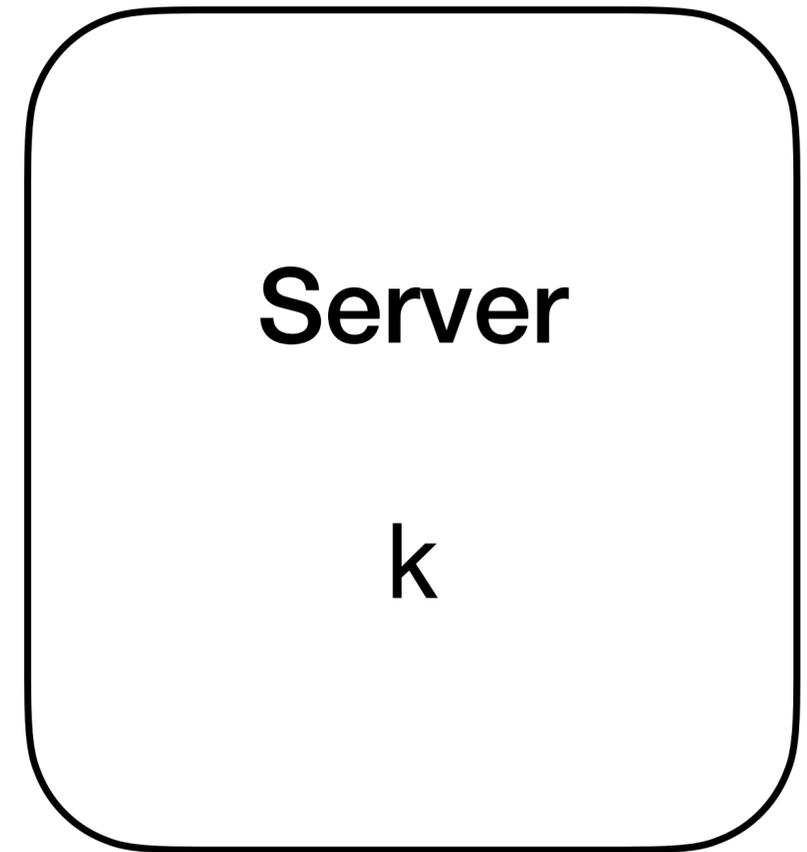
Counter (CTR) Mode

```
Enc(k, m_1 | ... | m_n):  
  r ← $ {0,1}^λ  
  for i in 1 to n  
    c_i ← F(k, r + i) ⊕ m_i  
  return r | c_1 | ... | c_n  
  
Dec(k, r | c_1 | ... | c_n):  
  for i in 1 to n  
    m_i ← F(k, r + i) ⊕ c_i  
  return m_1 | ... | m_n
```

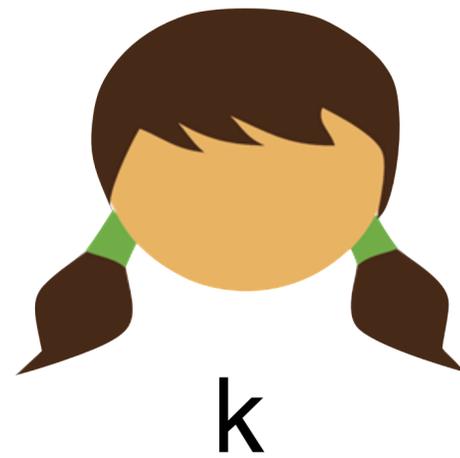




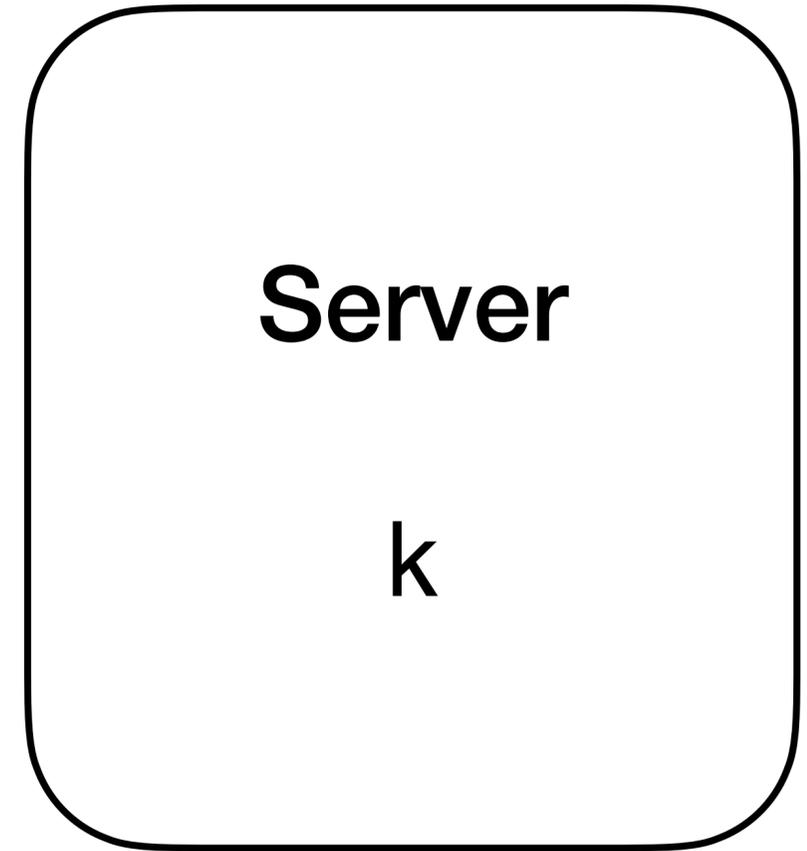
$$c = \text{CTR}(k, m)$$



$$m = \text{CTR}^{-1}(k, c)$$

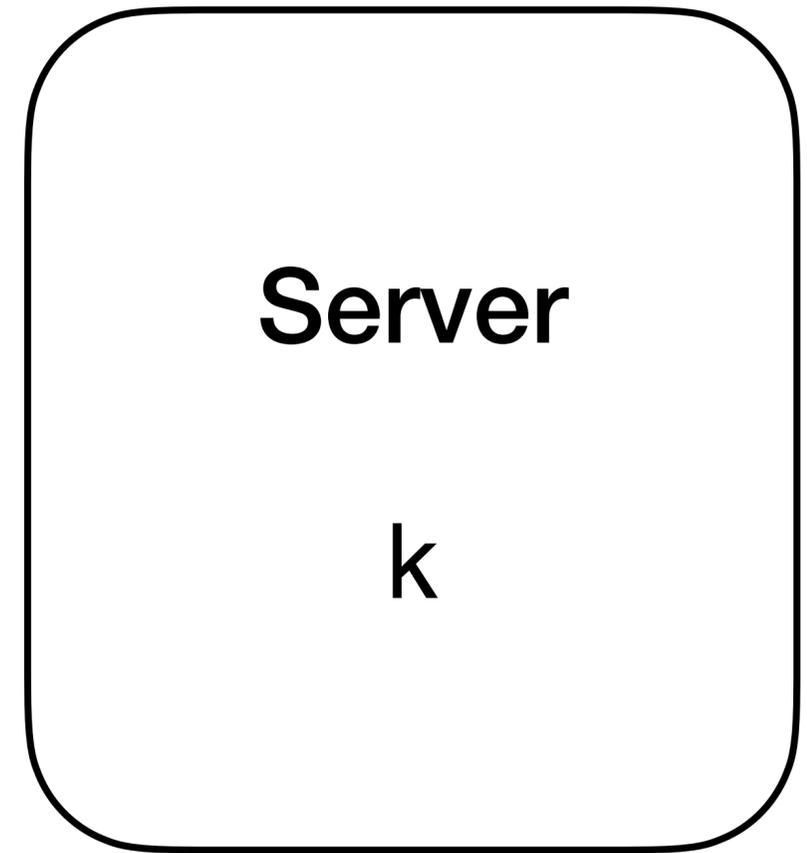
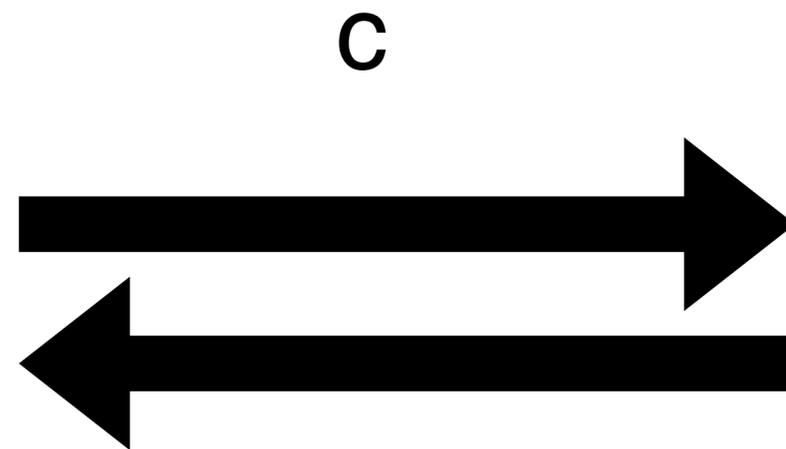
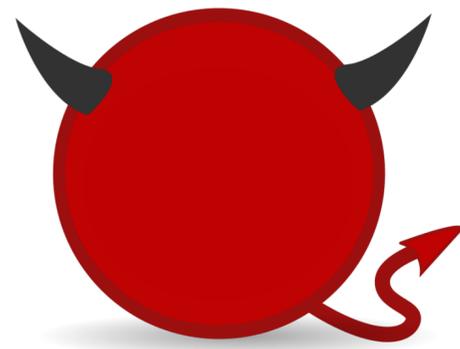


$$c = \text{CTR}(k, m)$$



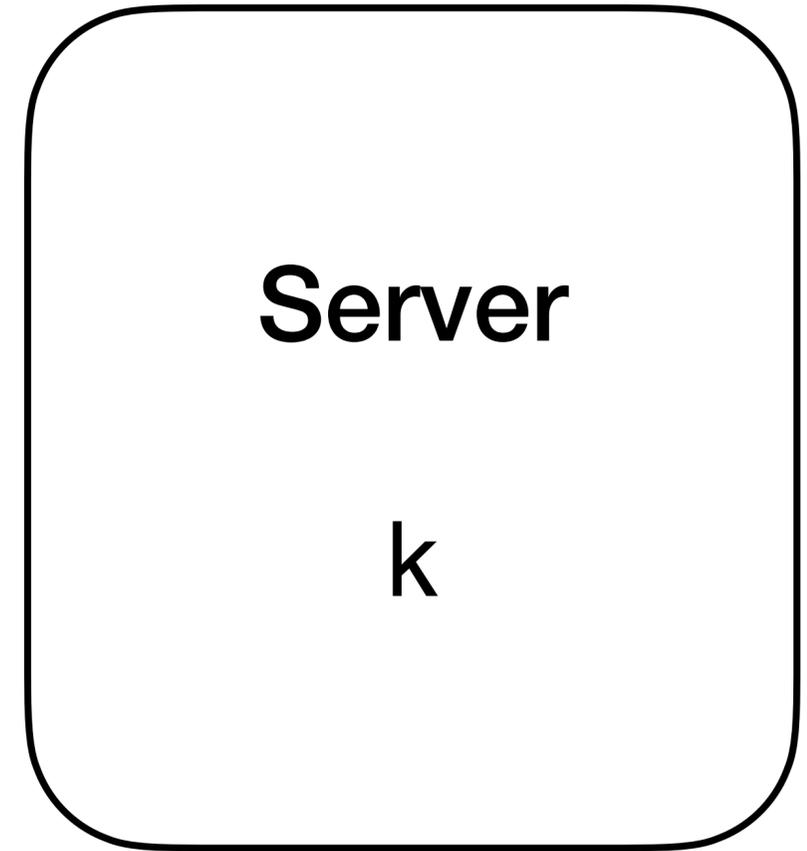
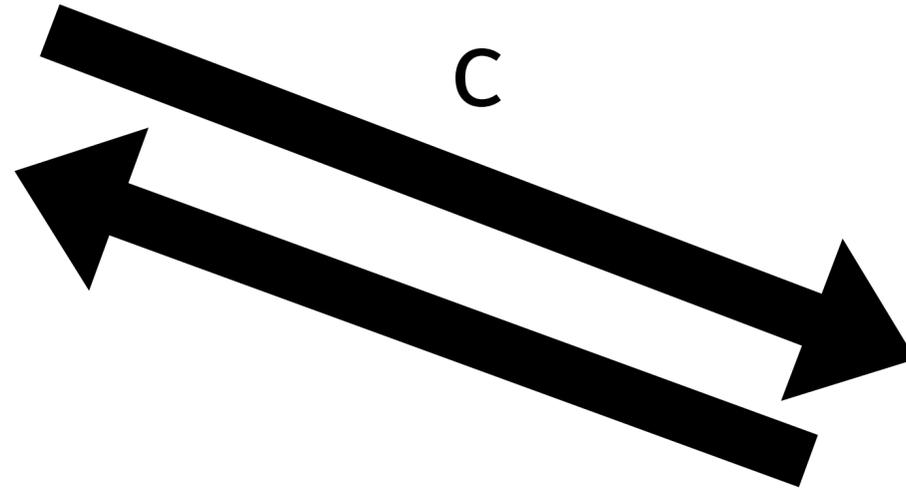
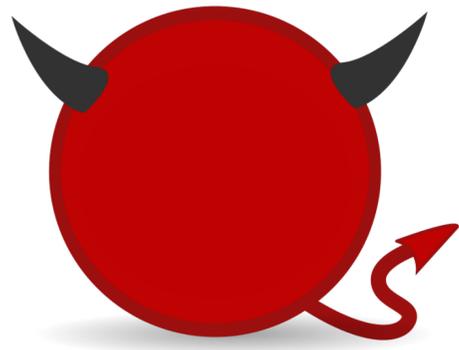
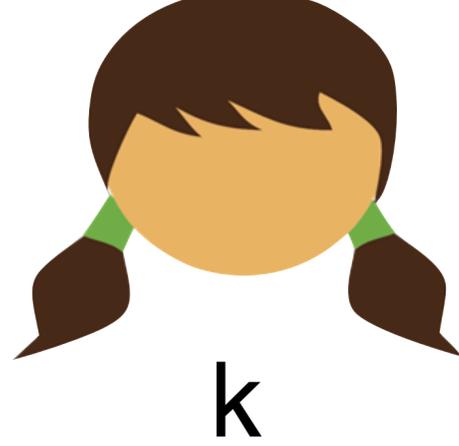
If m contains
 0×00 , ERROR

$$m = \text{CTR}^{-1}(k, c)$$



If m contains
 0×00 , ERROR

$$m = CTR^{-1}(k, c)$$



If m contains
 0×00 , ERROR

$$m = CTR^{-1}(k, c)$$

More Realistic: **Padding Attack**

Cipher Block Chaining (CBC) Mode

```
Enc(k, m_1 | .. | m_n):  
  c_0 ← $ {0,1}^λ  
  for i in 1 to n  
    c_i ← F(k, m_i ⊕ c_{i-1})  
  return c_0 | c_1 | .. | c_n
```

```
Dec(k, c_0 | c_1 | .. | c_n):  
  for i in 1 to n  
    m_i ←  $F^{-1}(k, c_i) \oplus c_{i-1}$   
  return m_1 | .. | m_n
```

Decrypting i-th ciphertext is possible from only a small part of the cipher text.

Padding:

$\text{pad}(m)$: takes input message, outputs string whose length is multiple of block length

$\text{unpad}(m)$: inverse of pad

Correctness: $\text{unpad}(\text{pad}(m)) = m$



Suggestion: Pad by a single 1, then pad with 0s until multiple of block length
To unpad, strip last 1 and all following 0s

Padding:

$\text{pad}(m)$: takes input message, outputs string whose length is multiple of block length

$\text{unpad}(m)$: inverse of pad

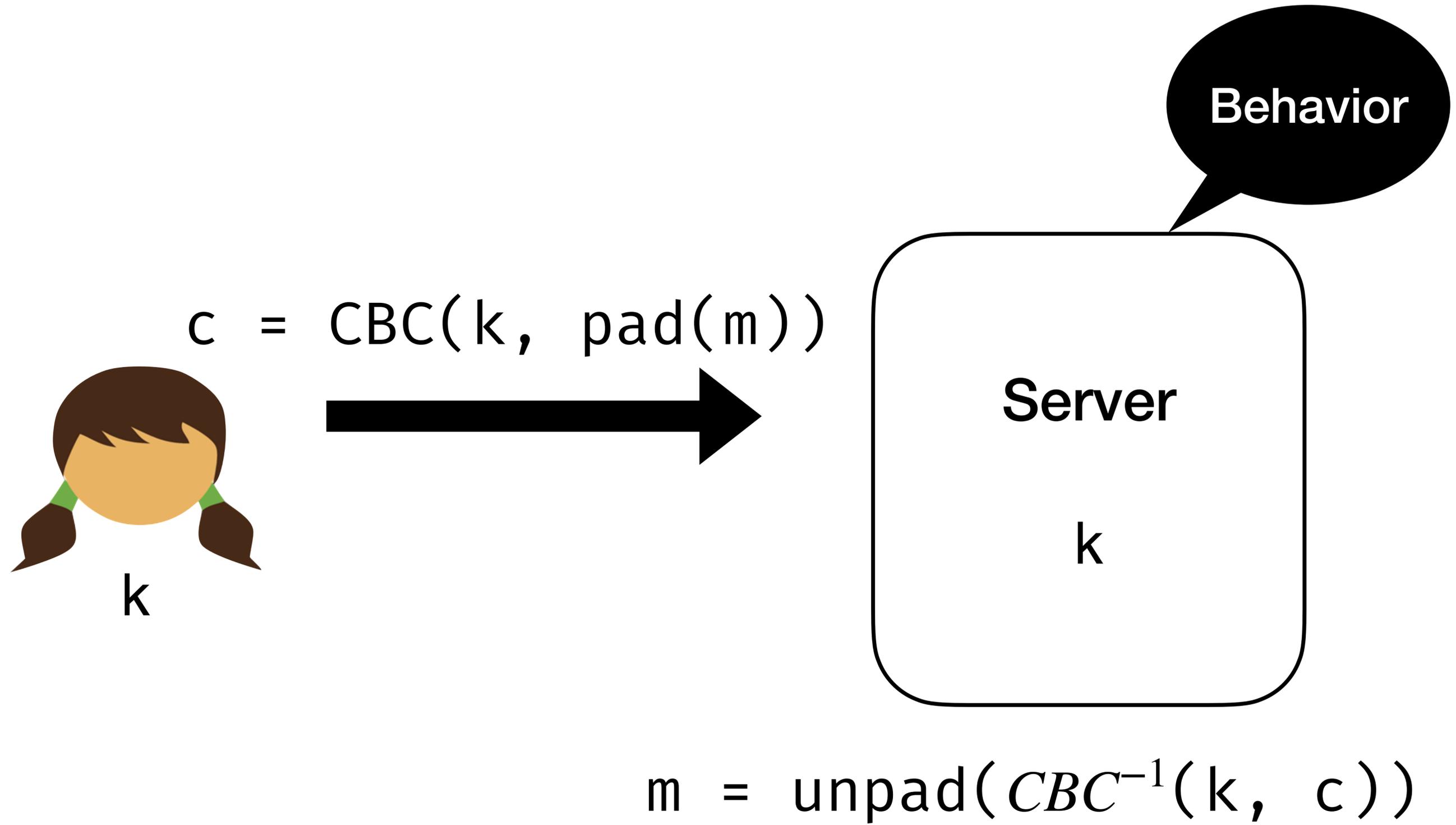
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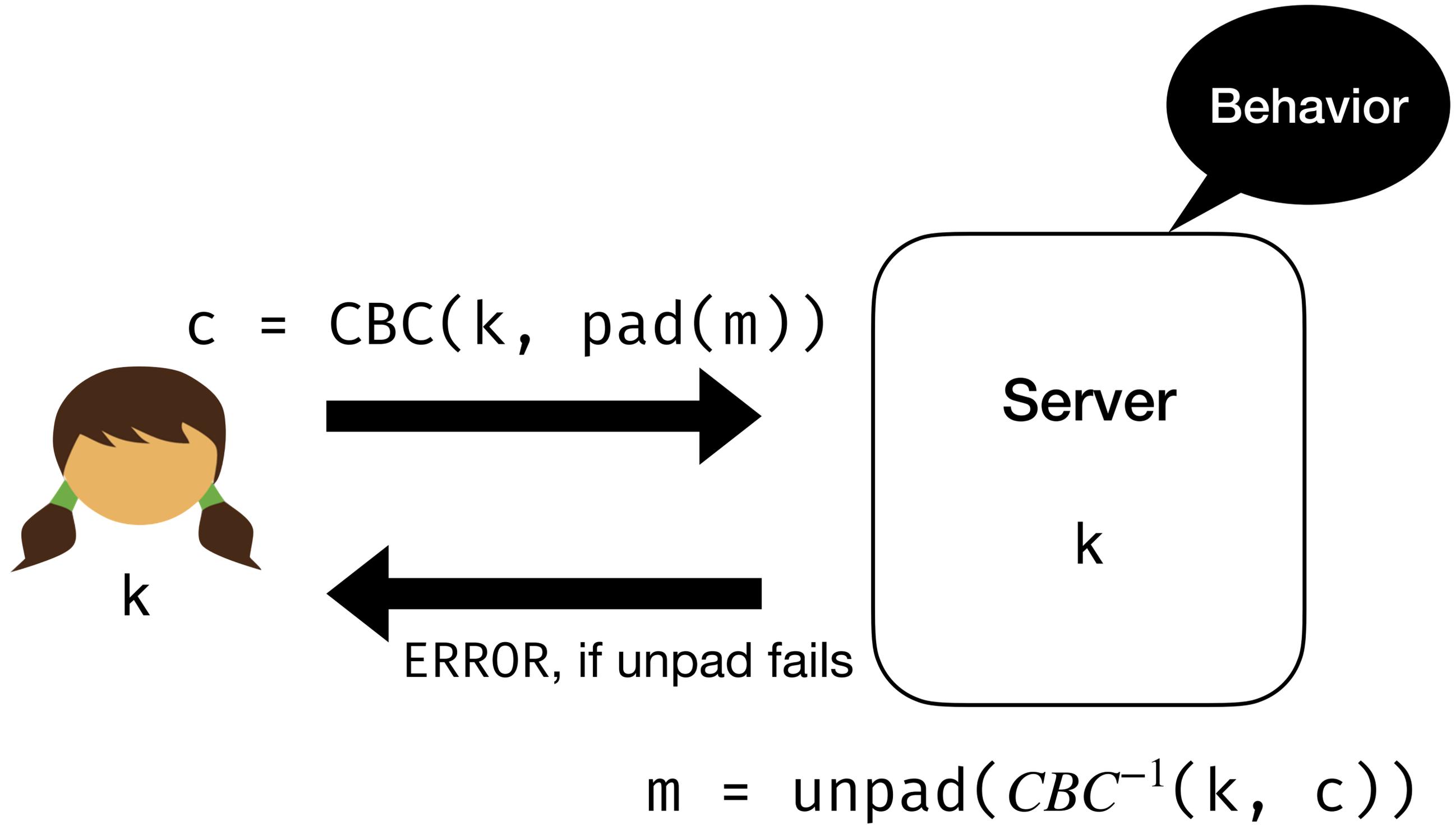


Suggestion: Pad by zeros, then write a single byte at the end that says how much padding was added.

To unpad, read last byte, check padding is **valid**, then strip that many bytes

X9.23 ANSI Padding Standard





Padding Attack

Why didn't CPA security protect us?

CBC Mode encryption is *malleable*:

Given ciphertext c encoding m , one can apply some function f to c such that $f(c)$ decrypts to m' , and the relationship between m and m' is *predictable*

Adversary's attack was on **decryption**, and CPA security says **nothing** about decryption

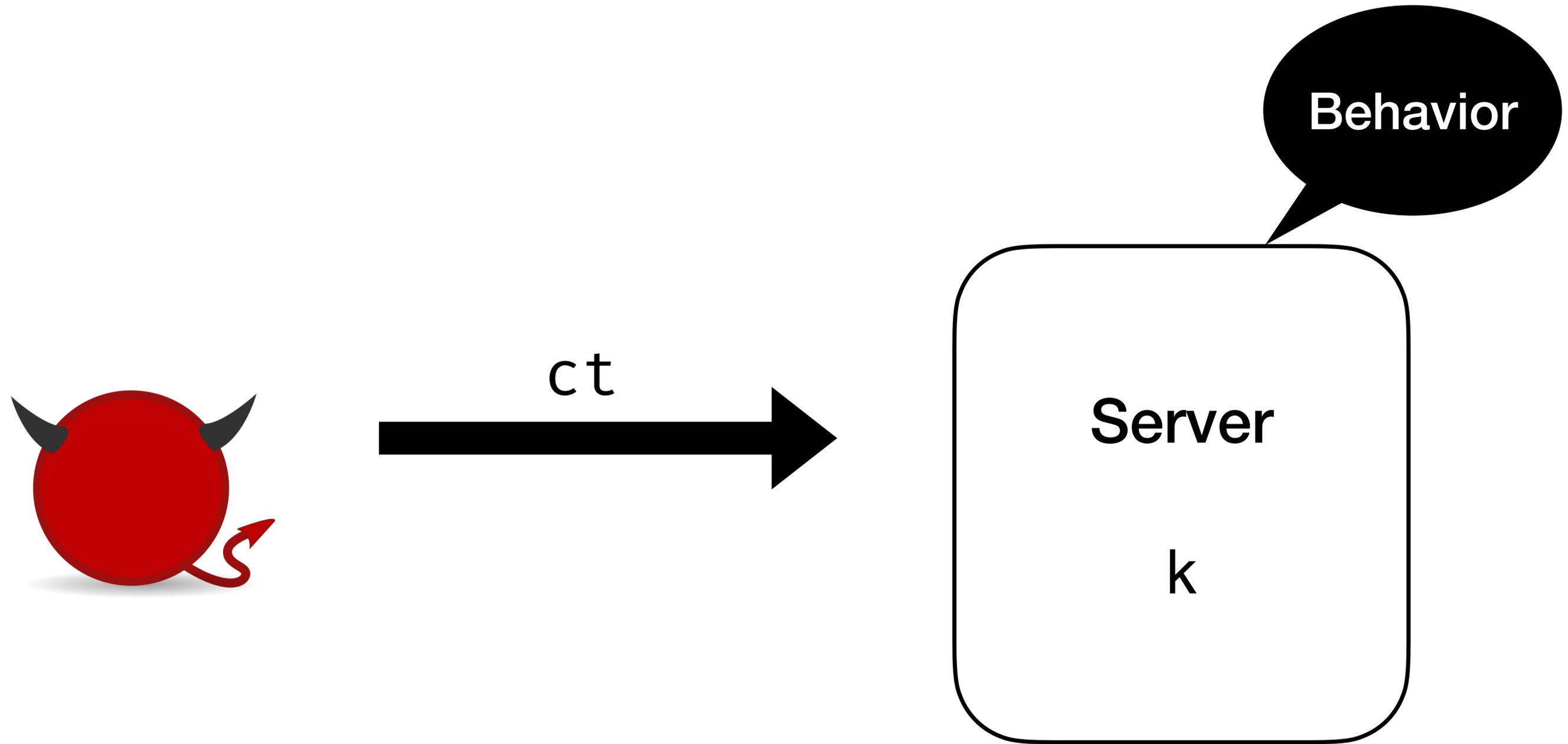
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Let $Enc_L(k, m_0, m_1) = Enc(k, m_0)$

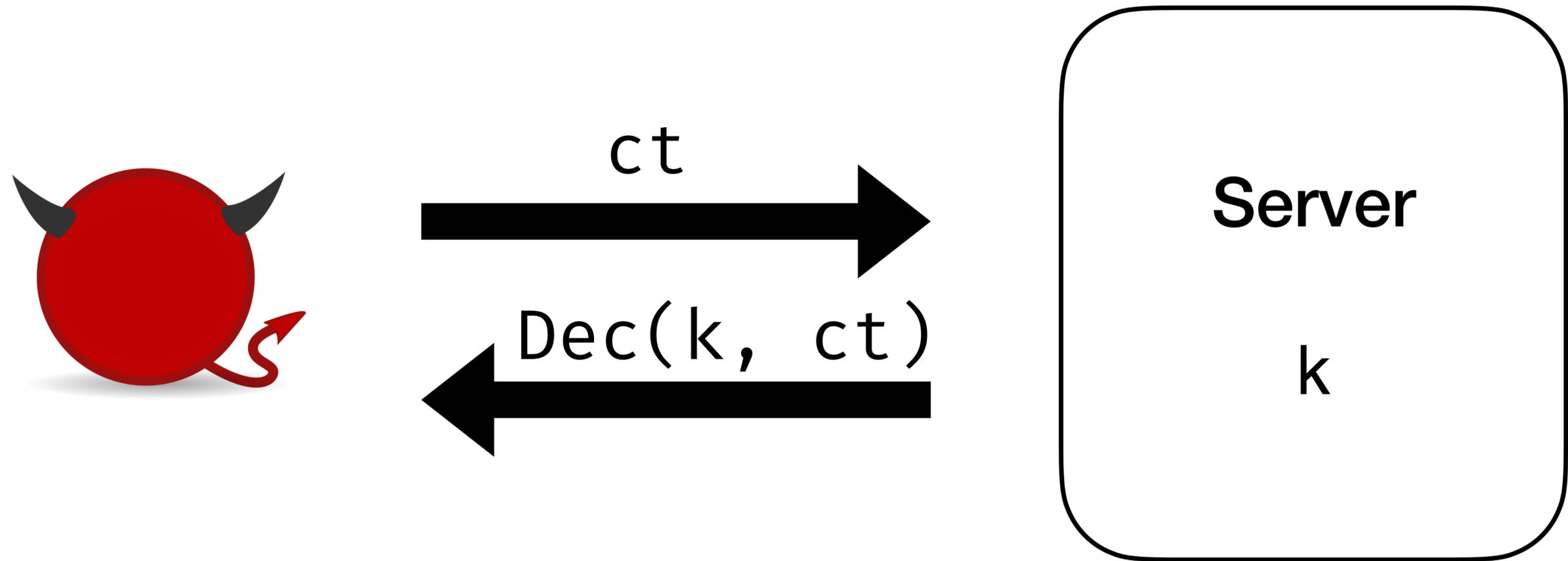
Let $Enc_R(k, m_0, m_1) = Enc(k, m_1)$

Where m_0, m_1 are of the same length

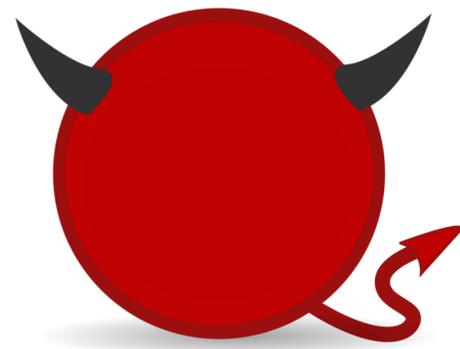
$$\left\{ Enc_L(k, \cdot, \cdot) \mid k \leftarrow K \right\} \approx \left\{ Enc_R(k, \cdot, \cdot) \mid k \leftarrow K \right\}$$



**security notion should capture
Adversary's ability to try ciphertexts**



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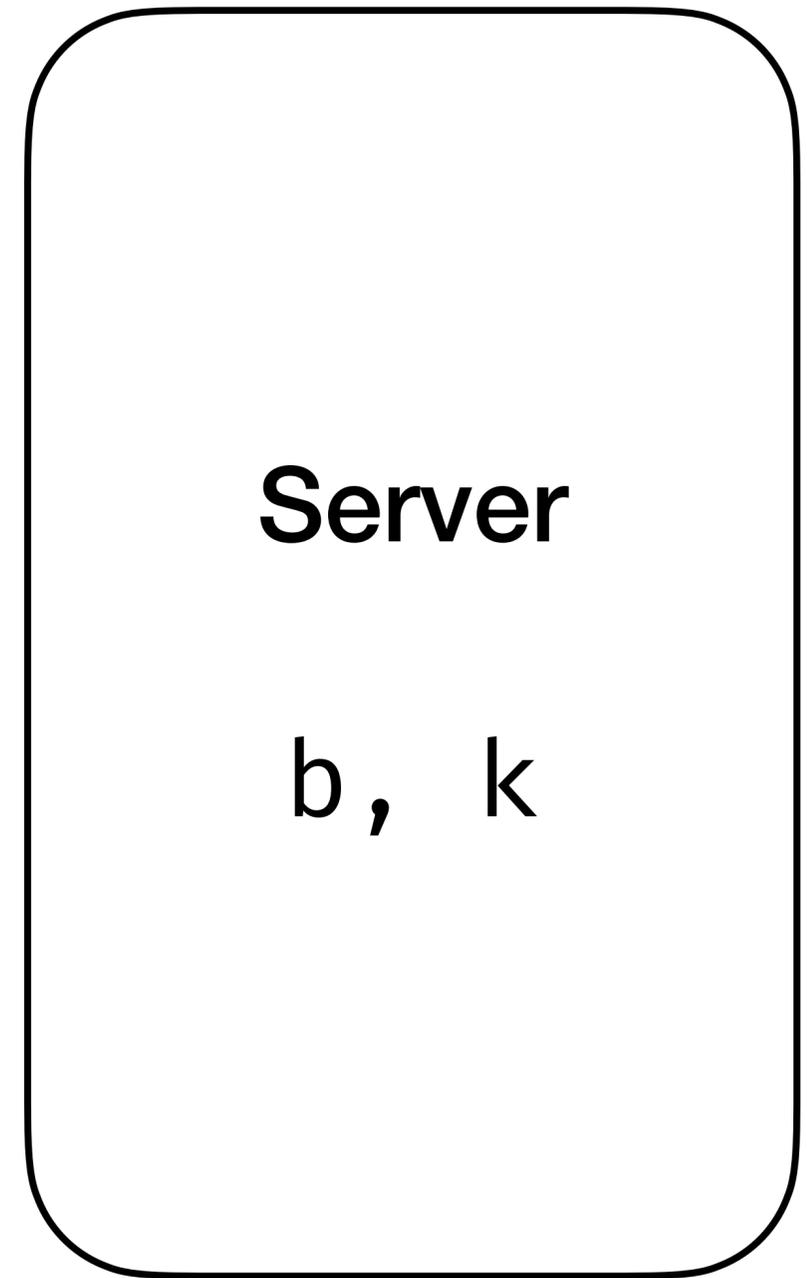


m_0, m_1

$\text{Enc}(k, m_b)$

ct

$\text{Dec}(k, ct)$



CCA Security

A cipher (Enc, Dec) has **security against a chosen plaintext attack (CPA)** if:

```
k ← $ {0,1}λ
```

```
encL(m0, m1):
```

```
  if |m0| ≠ |m1|:
```

```
    return error
```

```
  ct ← Enc(k, m0)
```

```
  return ct
```

≈

```
k ← $ {0,1}λ
```

```
encR(m0, m1):
```

```
  if |m0| ≠ |m1|:
```

```
    return error
```

```
  ct ← Enc(k, m1)
```

```
  return ct
```

A cipher (KeyGen, Enc, Dec) has **security against a chosen ciphertext attack (CCA)** if:

```
k ←$ {0,1}λ
S ← empty-set

encL(m0, m1):
  c ← Enc(k, m0)
  S ← S ∪ {c}
  return c

decrypt(c):
  if c ∈ S:
    return error
  return Dec(k, c)
```

≈

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Every CCA-secure scheme is also CPA-secure

Today's objectives

See intricate attacks by an active adversary

Understand the definition of **Security against Chosen Ciphertext Attacks (CCA)**

Explain why CTR/CBC mode block cipher is not CCA secure

Connect CCA security with **malleability**